COURSE : B . Sc (Hons) PHYSICS

CLASS : SEMESTER-6, SECTION-B

PAPER : ADVANCED MATHEMATICAL PHYSICS-II

TEACHER : SEEMA TRAMA

TOPIC : SUBGROUPS

The following are the REFERENCES for the 'GROUP THEORY' section of the syllabus:

M . Hammermesh : Group Theory and its Applications to Physical Problems.(MH)

A. W. Joshi : Elements of Group Theory for Physicists .(AWJ)

A.K. Vasishtha & A.R. Vasishtha: Modern Algebra(AKV)

READINGS FOR THIS TOPIC :

Chapter -1(MH) ; Chapter -1(AWJ) ; Chapter -2(AKV)

DEFINITION : A non-empty subset H of a group G is said to be a subgroup of G ,if H forms a group under the binary composition of G.

TRIVIAL/IMPROPER SUBGROUPS :

1) Every set is a subset of itself .So , if G is a group , then G itself is a subgroup of G.
2) If 'E' is the identity element of G ,then a subset consisting of only one element 'E' is also a trivial subgroup of G.

PROPER SUBGROUPS :

A subgroup other than the above two subgroups is called a proper subgroup of the group G.

NOTE : Here ,it is very important to note that a non-empty subset H of group G is a subgroup of G if the composition '*' in G is also the composition in H.(The composition is the same.)

LEARNING OBJECTIVES

After going through the above concepts from the readings you will be able to note :

1) Let (G,*) be a group . Then the subset H of G is a subgroup of G iff

a) $H \neq \emptyset$ that is , H is non-empty.

b) If a , b ϵ H ,then $a * b^{-1} \epsilon$ H

2) A set of problems is given below for your practice .

NOTE : In the problem set you are asked to determine whether the subsets of a given set are subgroups. One problem , is solved step-wise and in complete detail for your reference.

EXAMPLE : Let G be the additive group of integers . Then prove that the set of all integers multiplied by a fixed integer 'm' is a subgroup of G.

SOLUTION :G = {…………-3 , -2 , -1 , 0 , 1 , 2 , 3 , 4 , 5…………….}

G is the set of integers with the composition as addition . In this group we had already found that the additive identity = 0, and the inverse of each element 'a' is '-a'. (Example 1, study_material_4_Groups_1.pdf)

STEP 1 : Let 'm' be any fixed integer.

Then the set of all integers multiplied by the fixed integer 'm' is:

H={ ……………-3m ,-2m ,-1m , 0m , 1m , 2m , 3m……………..}

Note that all elements of H are multiples of 'm' AND they are also integers.

$H \neq \emptyset$ that is , H is non-empty.

STEP 2 : Let a , b ϵ H

Then let a = rm ϵ H (That is 'a' is some multiple of 'm')

Also ,let b = sm ϵ H (That is 'b' is some multiple of 'm')

Since a, b ϵ G (group) ,we know that the additive inverse of b is – b.

$\Rightarrow$ - b = - sm .

<u>Here we use the fact that the inverse of any element of a subgroup is the same as the inverse of the same element regarded as an element of the group.</u>

Now we find $a * b^{-1}$

$a * b^{-1}$ = a + (-b)

= rm – sm

= m (r - s)

Since r , s are integers , (r- s) is also an integer.

$\Rightarrow$ (r - s) m is also a multiple of m .

$\Rightarrow$ $a * b^{-1} \epsilon$ H

(Using the same binary composition "addition" we have proved that the element $a * b^{-1}$ obeys the same condition as the other elements of H ,that is, it is a multiple of 'm')

Therefore the subset H is a subgroup of G .

<u>METHOD 2</u> : Another way to check whether a subset is a subgroup is to check for all the group properties directly. If need be , you can construct the group multiplication table .

<u>PRACTICE PROBLEMS</u>

Check whether :

1) **The multiplicative group { 1, -1 } is a subgroup of the multiplicative group of fourth roots of unity: { 1,-1,i ,-i }.**
2) **The set Z – {0} , i.e. the set of non-zero integers is a subgroup of the group of non-zero rational numbers Q_0 over the composition of multiplication.**
3) **The group of even integers is a subgroup of the group of all integers under the composition of addition.**
4) **The group of positive rational numbers is a subgroup of all non-zero rational numbers under the composition of multiplication.**
5) **The group of integers is a subgroup of the group of rational numbers under the composition of addition.**
6) **i) Let G be the additive group of integers. Prove that the set of integers multiplied by a fixed integer 'm' is a subgroup of G.**

   ii) Show that the integral multiples of 5 form a subgroup of integers under the composition of addition.
7) **Let 'a' be an element of the group G . The set H is:**
   $H = \{ a^n : n \in Z\}$ of all integral powers of 'a' .Is H a subgroup of G under the composition of multiplication.