TUTORIAL - CUM ASSIGNMENT

Ques 1 Show that the set of all positive rational numbers from an abelian group under the composition defined by

$$a * b = (ab)/2$$

Ques 2. Show that the Set $G = \{a + b\sqrt{2} : a, b \in Q\}$ is a group with respect to addition.

Ques 3. show that set of all complex numbers of the form $\cos\theta + i\sin\theta$ where θ is any real number, forms a group with respect to the operation of multiplication of complex numbers.

Ques 4. Show that the set of matrices

$$A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

where α is a real number, forms a group under matrix multiplication.

Ques 5. Show that the set of all matrices $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, where a and b are real numbers not both equal to zero, is a group under matrix multiplication.

Ques 6. Show that the four fourth roots of unity namely $1, -1, i, -i$ from a group w.r.t to multiplication.

Ques 15. If a group G has four elements, show that it must be abelian.

Ques 16 If R is the additive group of real numbers and R_+ the multiplicative group of positive real numbers, prove that the mapping $f : R \to R_+$ defined by $f(x) = e^x \;\forall\; x \in R$ is an isomorphism of R onto R_+.

Ques 17 Let R_+ be the multiplicative group of all positive real numbers and R be the additive group of real numbers. Show that the mapping $g : R_+ \to R$ defined by $g(x) = \log x \;\forall\; x \in R_+$ is an isomorphism.

Ques 18 Show that the real matrices of the type $\begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}$, where $a \neq 0$ form a multiplicative group which is isomorphic to the group of non-zero real numbers under multiplication.

Ques 19 Show that the multiplicative group of all matrices $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, where a and b are real numbers is isomorphic to the group of non-zero complex numbers for multiplication.

Ques 7. Show that the set $G = \{1, \omega, \omega^2\}$, where ω is an imaginary cube roots of unity, is a group with respect to multiplication.

Ques 8 Show that the four matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

forms a multiplicative group.

Ques 9. Prove that the set of all n nth roots of unity forms a finite abelian group of order n with respect to multiplication.

Ques 10. Define a permutation. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$, find AB and BA.

Ques 11. Decompose the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 5 & 2 & 4 & 3 & 1 & 7 \end{pmatrix}$ into transpositions.

Ques 12 Show that the set P_3 of all permutations on three symbols 1, 2, 3 is a finite non-abelian group of order 6 w.r.t permutation multiplication as composition.

Ques 13 Show that the set G of four permutations I, (12)(34), (13)(24) and (14)(23) on four symbols 1, 2, 3, 4 is an abelian group w.r.t the permutation multiplication.

Ques 14 Find the order of each element in the multiplicative group $G = \{1, \omega, \omega^2\}$ where $\omega^3 = 1$.