

Course - B.Sc Physics (Hons)

①

Semester - IV

Paper Name - Mathematical Physics III

Name of the teacher - Dr. Surbhi Kumari

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Instructions to the Students

- ① I am continuing with my previous notes I shared with you that was the Properties of Fourier transform. Now you have to solve these numericals based on these properties.

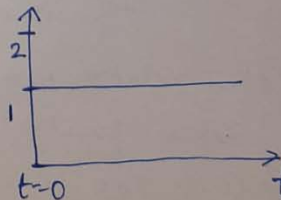
Problem I Based On Linearity Property

Q → Find the Fourier transform of a given signal

$$x(t) = \begin{cases} 2 & , t < 0 \\ 1 & , t > 0 \end{cases}$$

The signal is shown as

Here at $t=0$ the function value is 2 at $t > 0$ its value is 1 that is the unit step function.



Important → Here you have to study ^{first} the unit step function and its Fourier transform and other properties then you will be able to solve this problem.

② Q → find the Fourier transform of $x(t) = e^{-at} x(t) + u(t) + \delta(t)$ ②
Here $\delta(t)$ is an impulse function.

Important → You have to study the Fourier transform of an ~~delta~~ impulse function and its properties to solve this problem.

Problem II → Based on Shifting property or translation property.

1. Q → find the Fourier transform of $u(t-1)$.

Problem III → Based on Time Scaling property

1. Q → find the Fourier transform of $\delta(2t)$

2. Q → find the Fourier transform of $\text{Sgn}(2t)$.

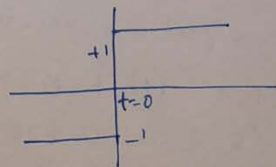
Here Sgn is called Signum function and is defined as

$$\text{Sgn} = \begin{array}{ll} -1 & t < 0 \\ 0 & t = 0 \\ 1 & t > 0 \end{array}$$

the Signum function is represented as

the relation between Signum function and unit step function is given by

$$\text{Sgn} = 2u(t) - 1.$$



Problem IV → Based on Fourier transform of derivative.

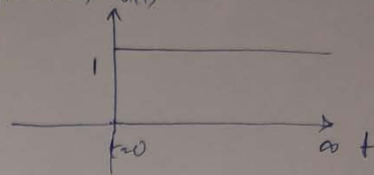
① Q → find the Fourier transform of Sgn function.
where the Sgn function is defined above.

Unit Step function →

③

Unit Step signal is given by $u(t)$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



Unit Step function

Q → Find the Fourier transform of $u(t)$ i.e. unit step function.

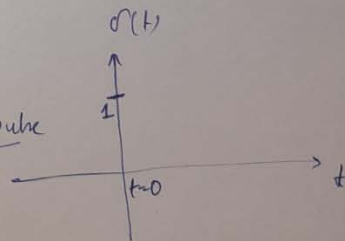
Important →

- ① If we multiply any function by $u(t)$ then its limit exists only for 0 to ∞ . Because unit step function is defined in the limit $0 \rightarrow \infty$.
- ② If we differentiate unit step function then we get the impulse function represented as $\delta(t)$.

Impulse function →

Unit Impulse function is given by $\delta(t)$

$$\delta(t) = \begin{cases} 1 & t=0 \\ 0 & t \neq 0 \end{cases}$$



Q → Find the Fourier transform of unit impulse function $\delta(t)$.

Important → $\int_{-\infty}^{\infty} \delta(t) dt = 1$

It means that if you have a unit impulse function then area covered by unit impulse is one. And if it is simply an impulse function then the area may varies.