

Tutorial #5

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Topic: Group Theory

Questions

- Q.1 Show that the four matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ form a multiplicative group.
- Q.2 Prove that the set of all n n^{th} roots of unity form a finite abelian group of order n with respect to multiplication.
- Q.3 Define a permutation, if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$, find AB and BA.
- Q.4 Find the inverse of each of the following permutations:
(i) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{bmatrix}$
- Q.5 Show that the set P_3 of all the permutations on three symbols 1, 2, 3 is a finite non-abelian group of order 6 with respect to permutation multiplication as composition.
- Q.6 Prove that the set A_3 of three permutations $(a), (abc), (acb)$ on three symbols a, b, c forms a finite abelian group with respect to the permutation multiplication.
- Q.7 Show that the four permutations $I, (ab), (cd), (ab)(cd)$ on four symbols a, b, c, d form a finite abelian group with respect to the permutation multiplication.