

Tutorial #4*Instructor: Ajay Mishra**Topic: Group Theory***Questions**

Q.1 Does the set \mathcal{I} of all integers form a group with respect to the binary operation \star defined on it as follows:

$$a \star b \rightarrow a + b + 1, \quad \forall a, b \in \mathcal{I}$$

Q.2 Show that the set \mathcal{C}_0 of all non-zero complex numbers in a group with respect to multiplication of complex numbers.

Q.3 Show that the set of all complex numbers of the form $\cos \theta + i \sin \theta$, where θ is any real number, forms a group with respect to the operation of multiplication of complex numbers.

Q.4 Show that the set of matrices

$$A_\alpha = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

where α is a real number, forms a group under matrix multiplication.

Q.5 Show that the set of all positive rational numbers (real numbers) forms an abelian group with respect to multiplication of numbers.

Q.6 Show that the four fourth roots of unity namely $1, -1, i, -i$ form a group with respect to multiplication.

Q.7 Show that the set $\mathcal{G} = \{1, \omega, \omega^2\}$, where ω is an imaginary cube root of unity, is a group with respect to multiplication.