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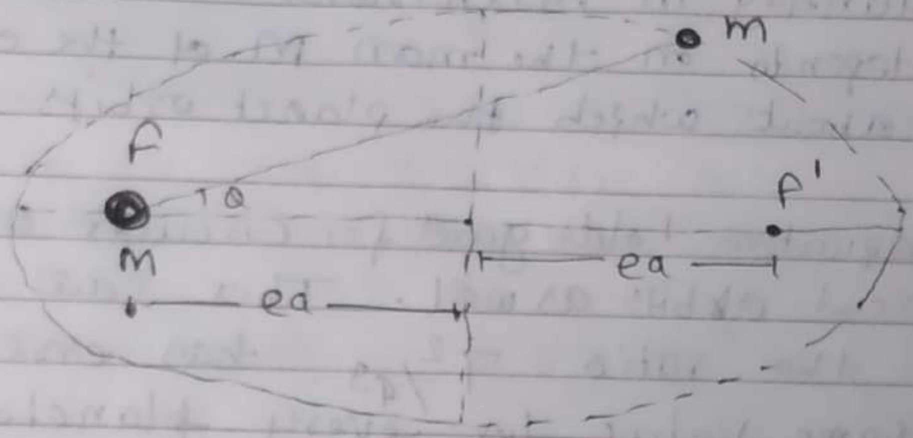
Kepler's laws

III. Johannes Kepler (1571-1630) after a lifetime of study worked out the Empirical laws that govern the motion of planets. Later Newton (1642-1727) showed that this laws of gravitation leads to Kepler's laws.

These laws applies to planets orbiting the sun and holds equally well for satellites, either natural or artificial, orbiting earth or any other massive central body.

First law: The law is called the laws of orbits and it says that all planets move in elliptical orbits, with sun at one focus.



A planet of mass m moving in an elliptical orbit around the sun. The sun of mass M, is at one focus F of the ellipse. The other focus is F' which is located in empty space. Each focus is a distance ea from the ellipse's center with e being eccentricity of the ellipse.

2: <u>The law of areas:</u> A line connects a planet to the sun sweeps out equal areas in the plane of planet's orbit in equal times, that is, the rate dA/dt at which it sweeps out area A is constant.

3: <u>The law of periods:</u> The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

$$T^2 = \left(\frac{4\pi^2}{GM}\right) a^3$$

The quantity in parentheses is constant that only depends on the mass M of the central body about which the planet orbits.

The equation holds good for circular and elliptical orbits as well. This laws predicts that the ratio T^2/a^3 has essentially the same value for every planetary orbit around a given massive body. For our solar system: $T^2/a^3 \simeq 2.99$ or 3.0

Planet	Semimajor Axis (10^{10} m)	Period (y) T	T^2/a^3
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.0	2.96
Saturn	143	29.5	2.98
Neptune	450	165	2.99

Satellites motion ::

As a satellite orbits Earth on its elliptical path both its speed, which fixes its kinetic energy K and its distance from the center of Earth, which fixes its gravitational potential energy U, fluctuate with fixed periods. However the mechanical energy E of the satellite remains constant.

(Since the satellite's mass is so much smaller than Earth's mass, we assign U and E for the Earth - satellite system to the satellite alone.)

The potential energy of the system is given by.

$$U = -\frac{GMm}{r} \quad \text{——} \quad (1)$$

(with U = 0 for infinite separation). r is the radius of the orbit, assumed for the time being to be circular and M, and m are the masses of Earth and the satellite, respectively.

To find the kinetic energy of a satellite in a circular orbit, lets use $f = ma$

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \text{——} \quad (2)$$

where $\frac{v^2}{r}$ is the centripetal acceleration of the satellite.

we can write eq (1) as

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r} \quad (3)$$

from eq (1) and (2)

The kinetic energy $K = -\frac{U}{2}$ (circular orbit)

Total mechanical energy of the orbiting satellite is

$$E = K + U$$

$$= \frac{GMm}{2r} - \frac{GMm}{r}$$

$$E = -\frac{GMm}{2r} = -K \quad \text{——— (3)}$$

This tells us that for a satellite in a circular orbit the total energy E is negative of the kinetic energy K.

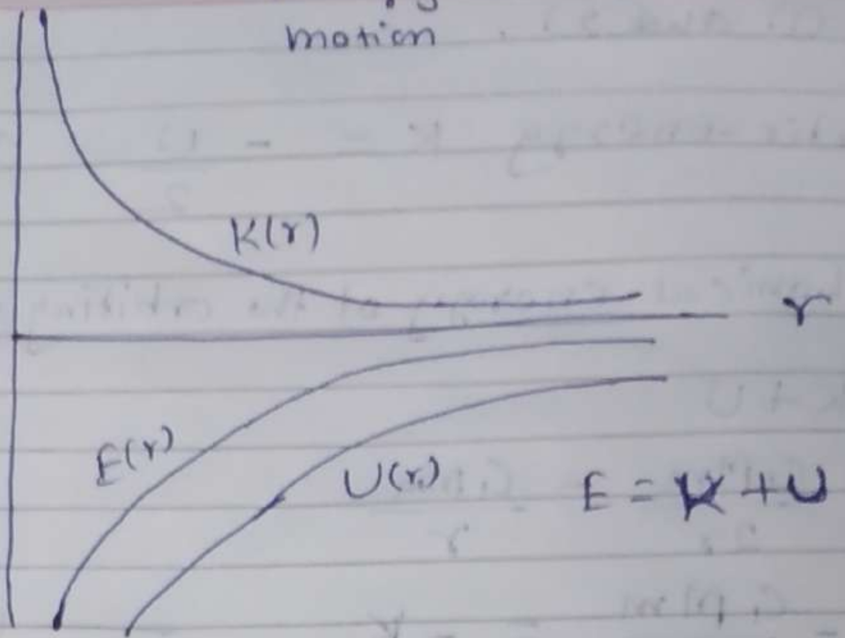
$$\boxed{E = -K} \quad \text{(Circular orbit)}$$

For a satellite in an elliptical orbit of semimajor axis a, we can substitute a for r in eq. (3)

$$E = \frac{-GMm}{2a} \quad \text{(elliptical orbit)}$$

The total energy of an orbiting satellite depends only on the semimajor axis of its orbit and on its eccentricity e.

Energy curve for a satellite in a circular motion



The variation of Kinetic energy K, potential energy U and total energy E with radius r for a satellite in a circular orbit. For any value of r, the value of U and E are negative, the values of K is positive and as $r \to \infty$ all three energy curves approach a value of zero.

Solve these Questions:

Q.1: A setellite is placed in a circular orbit about earth with a radius equal to one half the radius of moon's orbit. find the period of revolution of the setallite in lunar moths.

Q.N-2: The martian setallite phobos travels in an approx. circular orbit of radius 9.4×10^6 m with a period of 7h 35 min calculate the mass of Marse.

Q.3 An astronaut releases a ball of mass $m = 7.20$ kg into a circular orbit about earth at an altitude h of 350 km. find the mechanical energy of the ball in its orbit.

given - $G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$

Mass of earth $= 5.98 \times 10^{24}$ kg.

Radius of earth $= 6.72 \times 10^6$ m.