Date 18 03 2020

Teacher: Dr. Mahesh Chandra
Subject: Mechanics (GE)
Sem: IInd

Unit 4: Gravitation: Continued:

Let us first discuss about some definations of different type of forces which are important for motion of a particle in a central field.

1: Conservative force: If the total work done in moving a particle between two points is independent of the taken path then the force is consrvative.

Gravitational force is an example of a conservative force. The other examples are Electric (For ex. force on a charge particle in an electric field) and Spring force.

For a conservative force we can define a scalar potential at any point. So we can say when an object moves from one point to another; the force changes the potential energy of the object by an amout that does not depend on path taken.

2: Non-conservative force: where the work done does depend on the path taken, the force is nonconservative.

Ex: frictional force and drag forces.

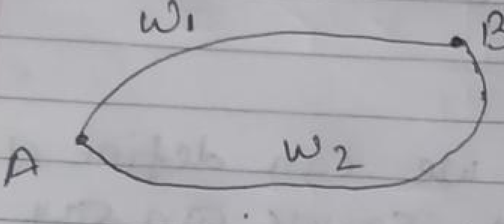
we can not define a potential for nonconservative forces because taking different paths would lead to conflicting potential differences between start and end points.

Remember that: The net work done by a conservative force on a particle moving around every closed path is zero.

3. <u>Central force:</u> If a force is directed towards a fix point and can be ~~der~~ derived from a scalar potential. then it is called a central force.

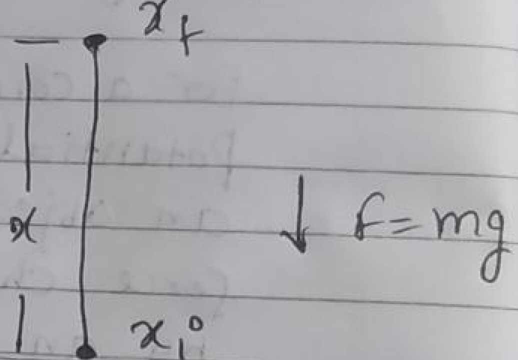
potential energy in gravitational force:

$$\Delta U = -W \qquad W - \text{work done by the force}$$



$W_1 = W_2$

Conservative force



If x_i and x_f are the initial and final position of a particle in a gravitational field.

Central force

$$\text{work done} = \int_{x_i}^{x_f} f \cdot dx$$

$$\Delta U = -\int_{x_i}^{x_f} (-mg) \cdot dx$$

$$= mg(x_f - x_i)$$

Change in potantial energy $\Delta U = mgx$

We can derive the force from this scalar potential:

$$\vec{F} = -\nabla U$$

where ∇U is the gradient of the potential energy.

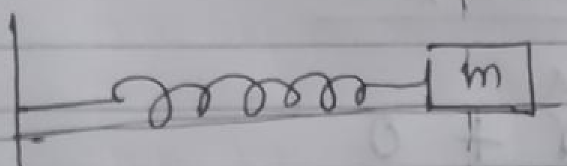
(we take gradient as U is scalar)

As force is directed towards a fix point we have only two directions +ve and -ve. So we can write $f = -\frac{dU}{dx}$

$$f = -\frac{d}{dx}(mgx) = -mg$$

Therfore the gravition force can be derived from a scalar potential.

similarly for spring force



$f = -kx$

$x=0$ The potential energy is given as $U = \frac{1}{2}kx^2$

which can give us the force

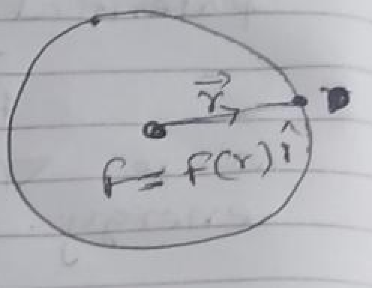
$$f = -\frac{d}{dx}U = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right)$$

$$\boxed{f = -kx}$$

Prove that in a central field the angular momentum is conserv

The angular momentum L is given by

$$L = \vec{r} \times \vec{p}$$



$\vec{p}$ is linear momentum given by

$$\vec{p} = m\vec{v} = m\frac{d\vec{r}}{dt}$$

So. $$\vec{L} = \vec{r} \times m\frac{d\vec{r}}{dt}$$

The change in angular momentum wrt time

$$\frac{dL}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

$$= \vec{r} \times \frac{d\vec{p}}{dt} + \vec{p} \times \frac{d\vec{r}}{dt}$$

$$= \vec{r} \times \vec{F} + \underline{m\vec{v} \times \vec{v}}$$

$$= |r|\hat{r} \times f(r)\hat{r} + 0$$

$$\boxed{\frac{dL}{dt} = 0} \Rightarrow \boxed{L = \text{constant}}$$

Hence the angular momentum in a centr force is conserved.