

## **MAGNETIC PROPERTIES OF SOLIDS**

### **INTRODUCTION**

Magnetism is perhaps that aspect of solid state physics which has been familiar to man for the longest period of time. The ability of lodestone (magnetite) to attract iron objects was known to ancient Greeks about 3000 years ago. A vast wealth of experimental data on the phenomenon was available long before any conclusive theory of magnetism could be developed. For example, it was known that iron could be magnetized by stroking it with another magnet, by hammering it when placed in the magnetic meridian, or simply by holding it somewhere in the vicinity of a strong magnet. Magnetic properties of solids originate in the motion of the electrons and in the permanent magnetic moment of atoms and ions. In other words, these are related to the spin and orbital angular momentum of electrons. Magnetic materials are technologically very important as they are used for data storage devices, preparing transformer cores, preparing magnets, and achieving low temperature. Before characterizing the magnetic properties of solids, we shall discuss spin and orbital magnetic moments.

### **THE ORBITAL MAGNETIC MOMENT**

The orbital magnetic moment of an electron arises in principle due to its orbital motion about the nucleus (see figure 1). A revolving electron in an orbit about the nucleus can be considered to be a small circulating current about the nucleus. This produces a magnetic moment  $\mu_{el}$  as shown in figure 1 given by

$$\mu_{el} = I A$$

where I is the current and A is the area of the orbit in which electron revolves.

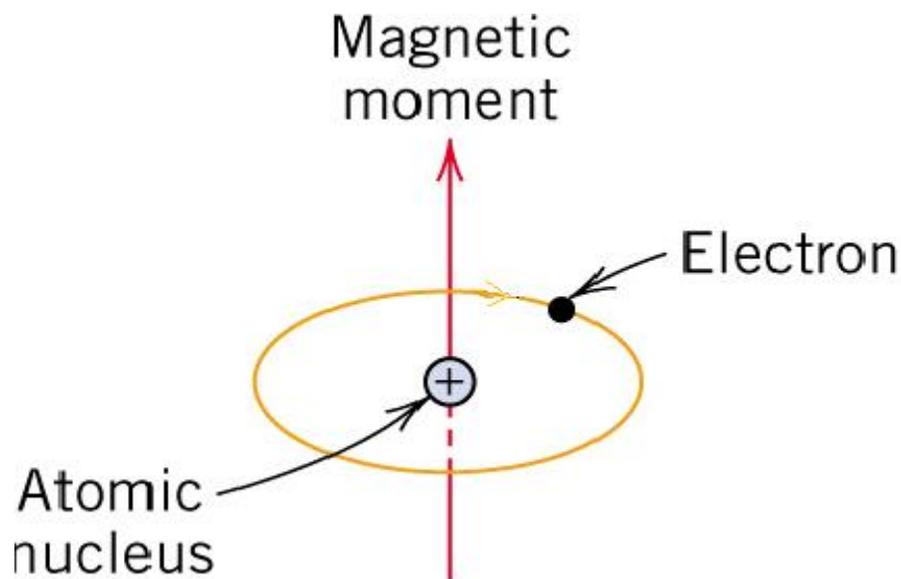
If we assume that electron moves in a circular path of radius  $r$  about the nucleus with a velocity  $v$ , then

$$I = \frac{e}{T}$$

But  $T = \frac{2\pi}{\omega}$

Also  $A = \pi r^2$

Thus  $\mu_{el} = \frac{er^2\omega}{2} = \left(\frac{e}{2m}\right)mr^2\omega$



**Figure 1 Orbital motion and corresponding magnetic moment of electron**

But  $mr^2\omega = L$  represents the angular momentum

$$\text{Thus } \mu_{el} = \left(\frac{e}{2m}\right) \text{Angular momentum} \quad (1)$$

The ratio of the magnetic dipole moment to the angular momentum of the electron due to its orbital motion is called orbital gyromagnetic ratio, given by

$$g = \frac{\text{Magnetic moment}}{\text{Angular momentum}} = \frac{e}{2m}$$

In quantum mechanics, the angular momentum of the orbit is determined by the quantum number  $l$  and is given by

$$L = \frac{h}{2\pi} \sqrt{l(l+1)} \text{ where } l = 0, 1, 2, \dots$$

An electron with  $l = 0$  i.e. s- electrons, always have zero angular momentum and thus zero magnetic moment. While other electrons associated with  $l = 1, 2, 3, \dots$  called p, d, f, ....electrons have non-zero angular momentum and thus have permanent magnetic moment, given by

$$\mu_{\text{el}} = \left( \frac{e}{2m} \right) \frac{h}{2\pi} \sqrt{l(l+1)} = \frac{e}{2m} L$$

or  $\mu_{\text{el}} = \mu_{\text{B}} \sqrt{l(l+1)}$

Where  $\mu_{\text{B}} = \frac{eh}{4\pi m}$  is called Bohr magneton and equal to  $9.27 \times 10^{-24} \text{ A-m}^2$ .

### THE SPIN MAGNETIC MOMENT

It is well known that electrons have spin motion in addition to their orbital motion; hence in addition to orbital angular momentum  $L$ , it has spin angular momentum  $S$ , which can be written as

$$S = \frac{h}{2\pi} \sqrt{s(s+1)},$$

where  $s$  is the spin quantum number and always has value  $s = 1/2$ .

The spin magnetic moment is associated with this angular momentum and expressed as

$$\begin{aligned}\mu_s &= g_s \sqrt{s(s+1)} \frac{eh}{4\pi m} \\ \mu_s &= g_s \sqrt{s(s+1)} \mu_B\end{aligned}\tag{3}$$

Here  $g_s$  is called the Lande splitting factor and for pure spin motion, its value is 2.

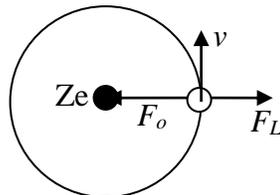
As we have already pointed out that an electron inside an atom is equivalent to a magnetic dipole. These electrons give rise to a net magnetic moment. This net magnetic moment may be zero or non zero depending upon the number of electrons in an atom. Moreover, the magnetic response of atoms and ions can show a variety of behaviour inside the materials depending upon the nature of interactions which occur between the magnetic moments produced by these atoms or ions. Keeping in mind this fact and the arrangement of magnetic dipoles, the magnetic materials can be classified broadly into four main categories, namely, diamagnetism, paramagnetism, ferromagnetism, and antiferromagnetism. The magnetic properties of solids are discussed in terms of magnetic susceptibility. Thus when a solid is placed in a magnetic field of intensity  $H$ , it is magnetized and a magnetic moment per unit volume,  $M$ , results.  $M$  is called magnetization. The magnetic flux density  $B$  is proportional to the intensity of the magnetic field, i.e.  $B = \mu H$ , where  $\mu$  is constant of proportionality, known as the permeability of the medium. For vacuum  $\mu = \mu_0$  and is equal to  $4\pi \times 10^{-7}$  henry/m. The ratio  $\mu / \mu_0 = \mu_r$  is called relative permeability of the medium. The degree of magnetization is generally expressed in terms of magnetic susceptibility  $\chi = M/H$ . As  $M$  and  $H$  have same units,  $\chi$  is dimensionless.

## DIAMAGNETISM

This effect is shown by those materials in which the constituent unit (atom, ion or a molecule) does not have unpaired electrons. In other words, we can say that the constituent units do not have net magnetic moment in the absence of magnetic field. This effect is caused by reaction of the orbiting electrons to an applied magnetic field in accordance with *Lenz's law*, so that magnetization and hence the susceptibility are both negative. Antimony, bismuth, mercury, gold, and copper are some examples of diamagnetic substances. These materials reduce the density of magnetic lines of forces. For these substances  $\chi < 0$  and it is independent of temperature.

### CLASSICAL THEORY OF DIAMAGNETISM (LANGEVIN THEORY)

Let us consider a diamagnetic material having  $n$  number of atoms per unit volume. Let  $Z$  be the atomic number. Let us assume that an electron revolves in anticlockwise direction (shown in figure) with an angular velocity  $\omega_o$  around the nucleus of charge  $Ze$  in a circular orbit of radius  $r$ . The Coulomb force of attraction between nucleus (charge  $Ze$ ) and electron provides the necessary centripetal force  $F_o$  to keep the electron in the orbit i.e.



$$F_o = m r \omega_o^2 = \frac{Z e^2}{4 \pi \epsilon_o r^2}$$

As the external magnetic field  $B$  is applied on the material in such a way that it is perpendicular to the plane of the orbit and in inward direction, an additional force  $F_L$  acts on the electron which is given by

$$F_L = -Bev = -Be\omega r$$

Thus the net force acting on the electron is

$$F = F_o - F_L = mr\omega_o^2 - Be\omega r$$

But  $F = mr\omega^2$  ; where  $\omega$  represents the angular velocity of electron after applying the magnetic field B

Thus 
$$mr\omega^2 = mr\omega_o^2 - Be\omega r$$

$$\Rightarrow \omega^2 = \omega_o^2 - \frac{eB\omega}{m}$$

or 
$$\omega^2 + \frac{eB\omega}{m} - \omega_o^2 = 0$$

$$\Rightarrow \omega = \frac{\left\{ -\frac{eB}{m} \pm \sqrt{\frac{e^2 B^2}{m^2} + 4\omega_o^2} \right\}}{2}$$

$$\Rightarrow \omega = \left\{ -\frac{eB}{2m} \pm \sqrt{\frac{e^2 B^2}{4m^2} + \omega_o^2} \right\}$$

$$\Rightarrow \omega = \left\{ -\frac{eB}{2m} \pm \sqrt{\frac{e^2 B^2}{4m^2} + \omega_o^2} \right\}$$

$$\Rightarrow \omega = \pm \omega_o - \frac{eB}{2m} \quad \text{if } \frac{eB}{2m} \ll \omega_o$$

The  $\pm$  sign on  $\omega_o$  indicates that those electrons whose orbital moments were parallel to the field are slowed down and those whose moments were antiparallel are speeded up by an amount  $\frac{eB}{2m}$ . This frequency change gives rise to a magnetization. The reason for this is that the frequency change is equivalent to an additional current, and this current component in every atom is in the same direction whereas the original circulating currents were in random directions and cancel each other. In the absence of a field, the electron motions being spherically symmetrical produce no net current and hence magnetic moment. But in the field, we can write a current for each electron due to its frequency change  $\frac{eB}{2m}$ . Now the change in the frequency of the electron is

$$\Delta\omega = \omega - \omega_o = -\frac{eB}{2m}$$

This change in frequency gives rise to a small value of magnetic moment  $\Delta\mu$  of the electron given by

$$\Delta\mu = \frac{er^2}{2} \Delta\omega$$

Putting the value of  $\Delta\omega$  in above equation, we get

$$\Delta\mu = -\frac{e}{2\pi} \pi r^2 \frac{eB}{2m} = -\frac{e^2 r^2 B}{4m}$$

Thus  $Z$  electrons give rise to magnetic moment  $\mu$  (say) given by

$$\mu = Z\Delta\mu = -\frac{Ze^2 B \langle r^2 \rangle}{4m}$$

If the orbit lies in the x-y plane, then

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle$$

If  $\langle r_o^2 \rangle$  represents the average distance of the electron from the nucleus, then

$$\langle r_o^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$$

For spherical symmetry  $\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle$

Thus  $\langle r^2 \rangle = 2 \langle x^2 \rangle$  and  $\langle r_o^2 \rangle = 3 \langle x^2 \rangle$

Hence  $\langle r^2 \rangle = \frac{2}{3} \langle r_o^2 \rangle$ ; putting this value we get:

$$\mu = Z\Delta\mu = -\frac{Ze^2 B \langle r_o^2 \rangle}{6m}$$

Thus magnetization  $M$  will be given by

$$M = n\mu = -\frac{nZe^2 B \langle r_o^2 \rangle}{6m} = -\frac{n\mu_o Ze^2 H \langle r_o^2 \rangle}{6m}$$

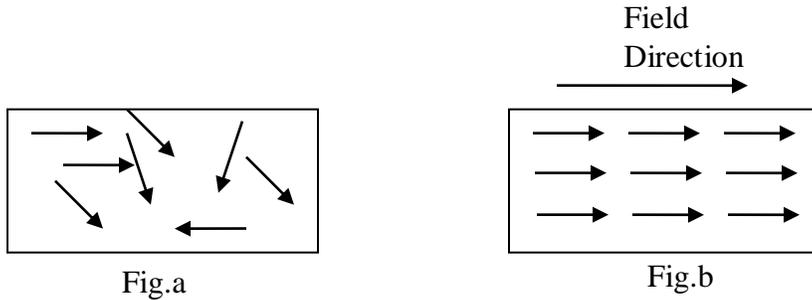
Thus

$$\chi = \frac{M}{H} = -\frac{n\mu_o Ze^2 \langle r_o^2 \rangle}{6m}$$

From the above equation it is clear that diamagnetic susceptibility  $\chi$  is always negative and is independent of temperature. It is a point to be noted that all materials exhibit diamagnetism though it is masked in case of para and ferromagnetic materials.

### PARAMAGNETISM

This effect is shown by those materials in which the constituent unit (atom, ion or a molecule) has unpaired electrons. In other words, we can say that the constituent atoms have a net magnetic moment in the absence of magnetic field. But the direction of these magnetic moments is random (shown in fig.a), as a result the net magnetic moment of the material is zero in absence of magnetic field as in case of diamagnetic materials. However, in the presence of the field, magnetic dipoles have a tendency to turn towards the direction of the field. If no opposing force acts, complete alignment of the magnetic dipoles will be produced (see fig.b) and the specimen as a whole would acquire a very large magnetization in the direction of applied field.



However thermal agitation of the atoms opposes the tendency and tries to keep the magnetic dipoles at random. This results in partial alignment in the field direction and hence small magnetization and susceptibility. Atoms and molecules having odd number of electrons (Al, Mg, Pt, transition elements, NO) are examples of paramagnetic

materials. Paramagnetism in general is a relatively small effect that has found a few technical applications *e.g.*, paramagnetic salts have been used in obtaining very low temperature of the order of  $10^{-3}$  K by adiabatic demagnetization.

**LANGEVIN THEORY OF PARAMAGNETISM**

The theoretical explanation for paramagnetism was given by Langevin using the classical statistics of Boltzmann. He assumed that each atom consists of a permanent magnetic moment  $\mu$  and that the only force acting on the atom is that due to the external field B. If a given atomic or molecular dipole is pointing in a direction making an angle  $\theta$  with B; its magnetic potential energy is,  $W = -\mu \cdot B = -\mu B \cos\theta$ . According to Boltzmann's classical statistics the number of atoms or molecules per unit volume having an energy in the range W and W + dW is

$$dn = Ae^{-W/kT} dW \dots\dots\dots(1)$$

where A is a constant to be determined, k is the Boltzmann constant and T is the absolute temperature.

On differentiating the potential energy expression, we get

$$dW = \mu B \sin\theta d\theta$$

Substituting the values of W and dW in equation (1) we get

$$dn = Ae^{\mu B \cos\theta/kT} \cdot \mu B \sin\theta d\theta$$

Now we shall integrate this expression over all possible orientations i.e. for  $\theta$  from 0 to  $\pi$  in order to find the total number of atoms or molecules n per unit volume of the substance. Thus

$$n = \int_0^\pi A e^{\mu B \cos \theta / kT} \cdot \mu B \sin \theta d\theta$$

Let  $\mu B / kT = x$  and  $\cos \theta = y$

$$n = -A \mu B \int_1^{-1} e^{xy} dy$$

This gives  $A = \frac{nx}{\mu B(e^x - e^{-x})}$  .....(2)

The component of each dipole moment parallel to B is  $\mu \cos \theta$  (see fig.1) and so the resultant magnetic moment due to all the  $dn$  atoms or molecules along the field is  $dn \mu \cos \theta$ .

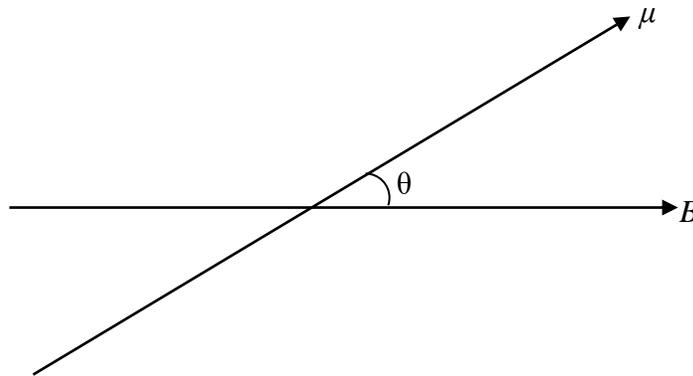


Fig.1

The magnetization  $M$  which is the magnetic moment per unit volume is thus given by

$$\begin{aligned}
M &= \int_0^\pi \mu \cos \theta . dn \\
&= A \int_0^\pi \mu^2 B e^{\mu B \cos \theta / kT} . \sin \theta \cos \theta d\theta \\
&= A \mu^2 B \int_0^\pi e^{\mu B \cos \theta / kT} . \cos \theta \sin \theta d\theta
\end{aligned}$$

Let  $\mu B / kT = x$  and  $\cos \theta = y$  then above equation becomes

$$\begin{aligned}
M &= A \mu^2 B \int_{-1}^{+1} e^{xy} y dy \\
&= A \mu^2 B \left[ \frac{ye^{xy}}{x} - \frac{e^{xy}}{x^2} \right]_{-1}^{+1} \\
&= A \mu^2 B \left[ \frac{e^x + e^{-x}}{x} - \frac{e^x - e^{-x}}{x^2} \right] \dots \dots \dots (3)
\end{aligned}$$

Putting the value of A from equation (2) in equation (3), we get:

$$M = n\mu \left[ \frac{e^x + e^{-x}}{e^x - e^{-x}} - \frac{1}{x} \right]$$

The quantity inside the square bracket is known as Langevin function

i.e. 
$$L(x) = \left[ \frac{e^x + e^{-x}}{e^x - e^{-x}} - \frac{1}{x} \right]$$

$$\therefore M = n\mu L(x)$$

The value of Langevin function depends on the value of x, when x is very large i.e. for high value of magnetic field B,  $L(x) = 1$  then  $M = n\mu = M_s$  known as saturated value of magnetization.

Thus  $M = M_s L(x)$  .....(4)

Thus Magnetization  $M = M_s$  i.e. saturation is reached when all atomic or molecular dipoles are parallel to B.

For small values of B, the value of x will be small, it can be easily proved that  $L(x) = x/3$  as explained below. Thus from equation (4)

$$M = M_s \frac{x}{3}$$

$$M = n \mu^2 B / 3kT$$

Or  $M = n \mu^2 \mu_0 H / 3kT$  .....(5) ( as  $B = \mu_0 H$ )

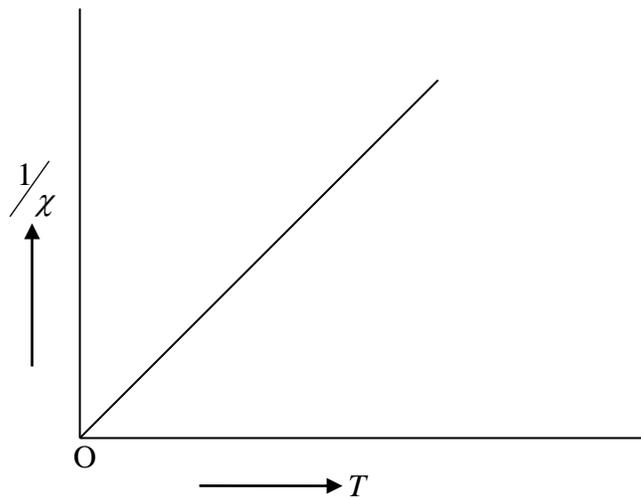


Fig.2

The susceptibility for a paramagnetic substance is thus given by

$$\chi = M/H = n \mu^2 \mu_0 / 3kT$$

The above expression can be written as

$$\chi = \frac{C}{T} \text{ .....(6)}$$

Where  $C = \frac{n\mu^2\mu_o}{3k}$  is called curie constant.

From equation (6), it is clear that the paramagnetic susceptibility  $\chi$  is inversely proportional to temperature. The variation of susceptibility with temperature is shown in Fig.2

### **FAILURE OF LANGEVIN THEORY**

1. It could not explain the complicated dependence of susceptibility upon temperature exhibited by several paramagnetics such as highly compressed and cooled gases, very concentrated solutions of salts, solid salts and crystals. Such substances obey the modified law

$$\chi = C/T - \theta, \quad \theta \text{ being a constant}$$

2. It could not explain the intimate relation between para and ferromagnetism.

### **WEISS THEORY OF PARAMAGNETISM**

Weiss in 1907 modified the Langevin theory of paramagnetism by introducing a new concept of internal molecular field. Weiss assumed that in paramagnetic substance, there must exist a molecular field produced at any point by all the neighboring molecules which is proportional to the magnetization vector i.e.

$$\text{Molecular field} \propto M = \alpha M$$

Thus the actual field acting upon a dipole is the sum of the applied field  $H$  and that arising from the presence of neighbouring dipoles, i.e. effective magnetizing field may be expressed as

$$H_i = H + \alpha M \quad \dots\dots\dots (7)$$

Explicitly assuming that the contribution of the neighboring dipoles to the effective field is proportional to the magnetic polarization. This form of dependence is not self evident

but was chosen by Weiss because it leads to the desired form. The factor of proportionality  $\alpha$  is known as the molecular field coefficient.

Now replacing  $H$  by  $H_i$  in (5) and substituting its value from (7) in equation (5)

$$M = n \mu^2 \mu_o H_i / 3kT$$

$$\therefore M = n \mu^2 \mu_o (H + \alpha M) / 3kT$$

$$= n \mu^2 \mu_o (H + \alpha M) / 3kT$$

So the Susceptibility

$$\chi = M/H = n \mu^2 \mu_o (H + \alpha M) / 3kTH$$

$$= \frac{n \mu^2 \mu_o}{3kT} + \frac{n \alpha \mu^2 \mu_o \chi}{3kT}$$

$$\Rightarrow \chi \left( 1 - \frac{\alpha n \mu^2 \mu_o}{3kT} \right) = \frac{n \mu^2 \mu_o}{3kT}$$

$$\chi = \frac{\frac{n \mu^2 \mu_o}{3kT}}{1 - \frac{\alpha n \mu^2 \mu_o}{3kT}}$$

$$\chi = \frac{\mu_o n \mu^2 / 3k}{T - \frac{\alpha \mu_o n \mu^2}{3k}}$$

$$\chi = \frac{C}{T - \Theta} \dots\dots\dots(8)$$

$$\frac{1}{\chi} = \frac{T - \Theta}{C} \dots\dots\dots(9)$$

where  $\frac{n \mu^2 \mu_o}{3k} = C$  known as Curie constant and  $\Theta = \frac{\alpha n \mu^2 \mu_o}{3k} = \alpha C$  is called

paramagnetic curie temperature. Thus equation (9) also represents the straight line having negative intercept.

Equation (8) represents the required Curie - Weiss law. One might predict from Curie - Weiss law that below Curie temperature the susceptibility of an otherwise paramagnetic material would be negative i.e., a paramagnetic material would become diamagnetic below Curie temperature. This prediction is wrong because it overlooks the fact that the analysis leading up to equation (5) is predicated upon the existence of high temperatures. Thus Curie - Weiss law holds only for temperature  $T > \Theta$ .

## **FERROMAGNETISM**

Like paramagnetism, this effect is shown by those materials in which the constituent unit (atom, ion or a molecule) has unpaired electrons. In other words, we can say that the constituent atoms have a net magnetic moment in the absence of magnetic field. But unlike paramagnetism, ferromagnetism is a strong effect. In some materials like Fe, Co, Ni, and Gd there is a self- alignment of group of atoms carrying permanent magnetic moments in the same direction resulting a large magnetic moment. Normally these magnetic moments are randomly oriented giving rise to almost negligibly small magnetization in the absence of external magnetic field. But when these materials are placed in an external magnetic field, then these moments align themselves in the direction of field giving rise to a large magnetization. Spontaneous magnetization is the most important feature of the ferromagnetic materials i.e. these materials show magnetic polarization and hence magnetization even after the removal of external field. It is a point to be noted that the spontaneous magnetization can occur only below a certain

temperature called ferromagnetic Curie temperature. Well above the Curie temperature, such materials behave like paramagnetic materials and have a well defined susceptibility given by Curie-Weiss law

$$\chi = \frac{C}{T - \Theta}, \text{ here } \Theta \text{ is ferromagnetic Curie temperature}$$

It has found experimentally that the existence of partially filled  $d$  or  $f$  is essential for ferromagnetism according to modern theories. Ferromagnetism arises from the alignment of electron spins due to spin exchange interaction throughout the solid. This occurs for partially filled bands with a high density of states near the Fermi level

## **DOMAIN THEORY OF FERROMAGNETISM**

Weiss in developing the theory of ferromagnetism introduced the new concept of magnetic domains. Like paramagnetic substances, the atoms of ferromagnetic substances have a permanent magnetic moment. But in ferromagnetic substances, the atoms, due to certain mutual interaction, form the innumerable small effective regions called *domains*. Each domain has  $10^{17}$  to  $10^{21}$  atoms whose magnetic axes are aligned in the same direction even in the absence of any external field. Each domain is in state of magnetic saturation, i.e., it is a strong magnet. However, in the normal state of substance, the different domains are randomly distributed as shown in figure (a). Hence their resultant magnetic moment in any direction is zero.

When a ferromagnetic substance is placed in an external magnetic field, the magnetic moment or the magnetization of the substance increases in two different ways:

- (1) By the displacement of the boundaries of the domains:** Consider an unmagnetised specimen in which each domain is spontaneously magnetized. The direction of magnetization is shown by arrow mark as shown in figure. When the

substance is placed in external magnetic field, the domains which are oriented favourable with respect to the external field increase in size while those oriented opposite to the external field are reduced as shown in figure (b).

**(2) By the rotation of the domains:** In this case the domains rotate until their magnetic momenta align more or less in the direction of external magnetic field as shown in figure(c).

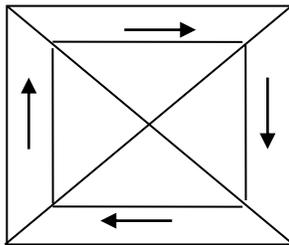


Figure (a)

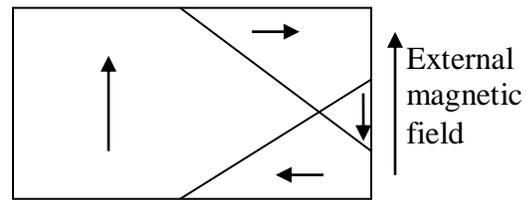


Figure (b)

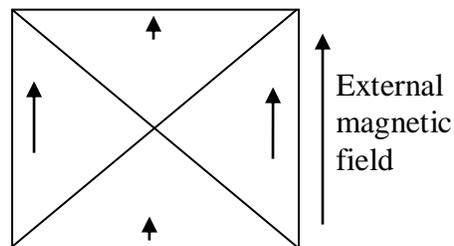


Figure (c)

It is a point to be noted that the substance is magnetized mostly by the boundary displacement when the external field is weak. On the other hand, in strong magnetic fields, the magnetization takes place mostly by the rotation of domains. When the domain vectors are oriented parallel to the direction of the external field, the stage is called as saturation limit. On the removal of external field, the boundaries do not move completely

back to their original position and hence the substance is not completely demagnetized, i.e., there still remains some residual magnetization. In other words, we can say that these materials show hysteresis. At high temperature, the domains are broken up and the ferromagnetic substance becomes paramagnetic.

### **WEISS THEORY OF FERROMAGNETISM (variation of spontaneous magnetization)**

It has been observed that transition metals, Fe, Co and Ni exhibit magnetism even when magnetisation field is removed. This phenomenon is known as ferromagnetism. To account for the special properties of ferromagnetics, Weiss gave a theory known as Weiss molecular field theory.

According to Weiss, the atomic *domains* magnets of ferromagnetic substances are grouped into certain region or *domains*. When the substance is in the unmagnetised condition, the domains form closed chains with no free poles. When the substance is magnetised, the chains break up and domains gradually set themselves with their magnetic axes all pointing in the field direction. Weiss suggested that the internal field experienced by a given dipole in ferromagnetic material is not only due to the applied field but also due to contribution from the neighbouring dipoles. This internal field tends to align the atomic dipoles in the same directions as its neighbours. The effective magnetic field  $H_i$  on the atomic dipole is

$$H_i = H + \alpha M \dots \dots \dots (1)$$

where  $\alpha$  is a constant and  $M$  = intensity of magnetisation.

$$H_i = H + \alpha M \dots \dots \dots (2)$$

Using Langevin's theory of paramagnetism, we have

$$M = n\mu L(x) \dots \dots \dots (3)$$

where  $n$  = number of dipoles per unit volume

$\mu$  = individual magnetic moment

$L(x)$  = Langevin's function =  $\coth x - 1/x$

Here  $n\mu$  is saturation magnetization, let it is represented by  $M_s$  and it corresponds to a situation when all the moments are aligned along the field direction.

$$M = M_s L(x) \quad \text{where } x = \frac{\mu B}{kT}$$

$$\frac{M}{M_s} = L(x) = (\coth x - 1/x) \dots \dots \dots (4)$$

Now,  $x = \frac{\mu B}{kT} = \frac{\mu\mu_o H_i}{kT}$  or  $x = \frac{\mu_o\mu}{kT} (H + \alpha M)$

But for ferromagnetic, spontaneous magnetization can take place even when  $H=0$

$$x = \frac{\mu_o\mu}{kT} (\alpha M)$$

or  $M = \frac{xkT}{\mu_o\mu\alpha}$

or  $\frac{M}{M_s} = \frac{xkT}{\mu_o\mu\alpha} \frac{1}{n\mu} \quad (M_s = n\mu)$

$$\frac{M}{M_s} = \frac{xkT}{n\mu^2\mu_o\alpha} \dots \dots \dots (5)$$

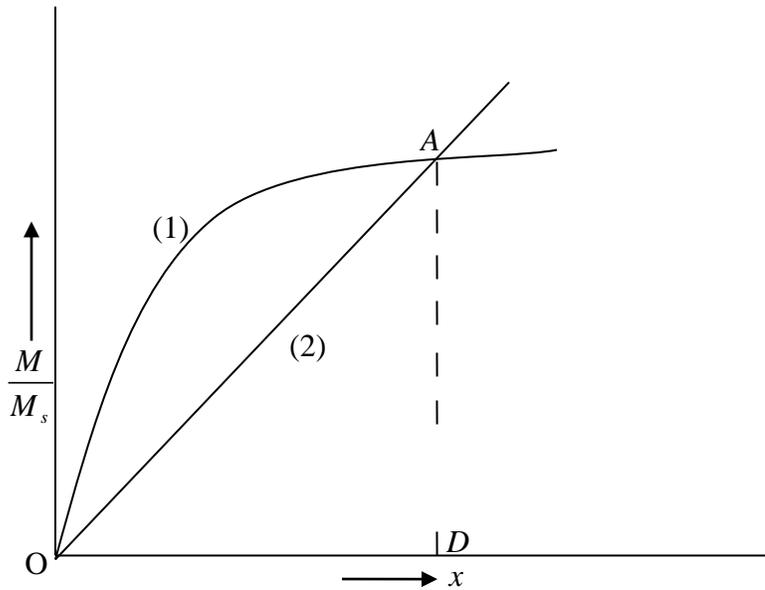
$$\Rightarrow \frac{M}{M_s} = \frac{3kT x}{n\mu^2\mu_o\alpha}$$

But  $n\mu^2\mu_o\alpha/3k = \Theta$  known as ferromagnetic curie temperature.

Thus 
$$\frac{M}{M_s} = \frac{T x}{3\Theta} \dots\dots\dots(6)$$

Thus plot of  $M/M_s$  versus  $x$  corresponding to equations (4) and (6) together determine the condition of spontaneous magnetization.

The figure shows the plot drawn between the two. Curve (1) is Langevin curve corresponding to equation (4) while straight line is corresponding to equation (6). When  $T = \Theta$  then slope of equation (6) will be  $1/3$  and the two curves will not intersect at non-zero point and hence there will be no spontaneous magnetization but when  $T < \Theta$  only then the two curves will intersect at non-zero point  $A$  and there will be spontaneous magnetization i.e. the material will show the ferromagnetic behavior only below ferromagnetic curie temperature  $\Theta$ . Hence  $M/M_s=AD$  corresponds to spontaneous magnetization.



It is obvious from the graph that  $A$  represents the state of spontaneous magnetization. Further if the temperature of the material will be increased from  $0\text{K}$  to ferromagnetic curie temperature then there will be a decrease in the spontaneous magnetization due to decrease in the slope of straight line equation corresponding to equation (6).